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**NAVORD REPORT** 2691    **RT** 2691

# THE EFFECT OF THE OPTIMUM CHAMBRAGE ON THE MUZZLE VELOCITY OF GUNS WITH A QUALITATIVE DESCRIPTION OF THE FUNDAMENTAL PHENOMENA OCCURRING DURING GUN FIRING

**I OCTOBER 1952**

# U. S. NAVAL ORDNANCE LABORATORY

## WHITE OAK, MARYLAND

# THE EFFECT OF THE OPTIMUM CHAMBRAGE ON THE MUZZLE VELOCITY OF GUNS WITH A QUALITATIVE DESCRIPTION OF THE FUNDAMENTAL PHENOMENA OCCURRING DURING GUN FIRING

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ABSTRACT: A qualitative description of the effect of chambrage, the increase in cross-sectional area in going from the barrel bore to the propellant chamber, is presented. Included in the discussion are the effects of the other fundamental factors which determine the pressure behind a projectile, vis., the burning of the propellant, the presence of the breech end, and the accelerating projectile motion. The description is given in terms of the rarefaction and compression impulses which are present in the propellant gas during the firing of a gun.

In ballistic calculations, chambrage is treated by assuming that the actual chamber can be replaced by an equal volume imagined chamber of cross-sectional area equal to that of the bore. From the qualitative description of chambrage, it is seen that this method of treatment may be in error. The possible size of this error is indicated by a calculation of the muzzle velocity of a gun with an optimum, most favorable condition of chambrage - a condition approached by a gun with either a large, well-shaped chamber or with a propellant burning at the proper rate in a smaller chamber. This calculation demonstrates that the muzzle velocity of the optimum chambrage gun is as much as 28 per cent greater than that of the comparable constant diameter gun.

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This report presents the results of a study of the effect of
chambrage (the increase in cross-sectional area in going from the
barrel to the chamber of a gun) on the muzzle velocity of guns.
It was carried out under the sponsorship of the Bureau of Ordnance
project NOL-Re5a-30-1-53.

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CONTENTS

APPENDIX

LIST OF FIGURES

THE EFFECT OF THE OPTIMUM CHAMBRAGE ON THE MUZZLE VELOCITY OF GUNS
WITH A QUALITATIVE DESCRIPTION OF THE FUNDAMENTAL PHENOMENA
OCCURRING DURING GUN FIRING

## I. INTRODUCTION

The effect on the muzzle velocity of guns of chambrage, the
increase in cross-sectional area in going from the barrel bore to
the propellant chamber, has not been quantitatively determined.
For such a determination it would be necessary to apply a numerical
characteristics method in two-space dimensions (axial and radial) to
the unsteady problem. However, until the present time this procedure
has not proved feasible.

In ballistic calculations, chambrage is universally treated by
assuming that the actual chamber can be replaced by an equal volume
imagined chamber of cross-sectional area equal to the bore cross-
sectional area. This assumption is employed because of the lack of
any other method of dealing with chambrage; experimental results of
gun firings are inconclusive as to its validity.

In this paper the effect of chambrage on the muzzle velocity of
guns is obtained for an optimum, most favorable condition of
chambrage. This condition of "optimum chambrage" is a limit which
is approached, but never exceeded, by modern guns.

Consideration is also given here to the factors other than
chambrage which determine the pressure behind a projectile, viz.,
the burning of the propellant, the presence of the breech end, and
the accelerating projectile motion. These are qualitatively
examined below. The effects of the irreversibilities (such as
gas-wall friction, gas viscosity, projectile friction) and the heat
transfer from the propellant gas to the gun barrel walls occurring
during firing are not discussed.

## II. A QUALITATIVE DESCRIPTION OF THE EFFECT OF
CHAMBRAGE ON MUZZLE VELOCITY

When the projectile in a gun begins to move, expansion (also
termed rarefaction) disturbances or impulses are sent back to the
breech. These rarefaction impulses are characterized by the fact
that they decrease the pressure and density of the gas through which
they pass (in contrast to compression impulses which increase the
pressure and density of the gas through which they pass). This
pressure drop results from the fact that gas is accelerated by the
rarefactions "shed" from the accelerating projectile; hence, this
pressure drop is designated here as the "pressure drop from the
accelerating projectile motion". In Appendix I this pressure drop
is discussed, and an analytic expression is given for it. During
the entire movement of the projectile in the barrel, the projectile
continues to shed these rarefactions which travel toward the breech
at the local velocity of sound of the propellant gas. Consequently,

1

the pressure of each portion of the gas behind the projectile drops
(or tends to drop) continuously as the projectile accelerates toward
the muzzle; in particular, the pressure of the gas portion directly
behind the projectile drops the most, since all of the rarefactions
travel through this gas portion initially.

In a gun with no chambrage, i.e., a constant cross-sectional area
gun, these impulses upon reaching the breech end are reflected as rare-
factions and travel up the barrel toward the back end of the projec-
tile, lowering the pressure of the gas through which they travel. When
they reach the back end of the projectile, these impulses, sent forward
because of the presence of the breech, lower the pressure behind the
projectile; consequently, the projectile's velocity is not as large as
it would be if these rarefaction impulses had not reached the projec-
tile (as in the case when the breech end is sufficiently distant from
the projectile that the projectile is out of the barrel before the
rarefactions reach it).

A more complex phenomenon occurs in a gun with chambrage, i.e., a
gun with a chamber whose cross-sectional area is larger than that of
the bore. In such a gun a part of each of the rarefaction impulses
shed from the back of the moving projectile is reflected as a
compression impulse at the section where the bore cross-sectional area
increases*; this compression impulse upon reaching the projectile tends
to raise the pressure behind the projectile and thus increases the pro-
jectile velocity. The remaining part of the rarefaction impulse con-
tinues its travel toward the breech still as a rarefaction; at the
breech it is reflected as a rarefaction impulse, and at the section of
area decrease a portion is reflected still as a rarefaction impulse,
while the remaining portion continues its travel toward the projectile
as a rarefaction impulse. This sequence of events continues as the
projectile moves along the barrel. Therefore, the projectile in a gun
with chambrage experiences as a result of the change in area section
and the presence of the breech a combination of compression and rare-
faction impulses. To reiterate, superimposed on the ever-present
pressure drop from the accelerating projectile motion is a further
pressure drop from the rarefactions sent forward as a result of the
presence of the breech and a pressure rise from the compressions sent
forward as a result of the increase of area section.

The rise in pressure of the gas behind the projectile as a result
of the compression impulses reflected from the change in area section
is determined by the "effectiveness" of these compressions and by the
amount of compressions that are reflected. The effectiveness of the
compressions depends on two factors. The first is the velocity of
the projectile. For a given projectile velocity there corresponds a
maximum rise in pressure possible by the action of the compression

*This is evident upon realising that the gas flowing from the larger
chamber to the smaller bore section as a result of the rarefaction
impulse from the projectile tends to crowd or compress the gas in the
barrel bore section.

impulses. (This follows from the energy conservation law; the principle stated loosely is that the gas causing the "push" on the projectile drops in internal energy and therefore in pressure in order to gain the necessary kinetic energy to "push".) This point is elaborated upon in Appendix II. The possible rise in pressure from the compression impulses becomes less and less as the projectile velocity increases. (See Appendix II.)

The second factor determining the effectiveness of the compression impulses is the thermodynamic state of the gas in the chamber at rest. Any of the thermodynamic properties of the gas at rest, such as enthalpy, sound velocity, or pressure can be used as the criterion here. If these thermodynamic stagnation properties drop in value during the firing of a gun, the subsequent compressive effect of the gas in the chamber also drops; if they rise, the compressive effect rises. (See Appendix II).

It is to be stated that in contrast to the effects of the reflected compressions the effects of the rarefaction impulses do not diminish either with increasing projectile velocity or decreasing thermodynamic stagnation state. The rarefactions traveling between breech and projectile will, if the barrel is sufficiently long, decrease the pressure behind the projectile to zero.

The amount of the compression impulses that are reflected from the change in area section is a function of the gun geometry. In a gun with chambrage, the greater the increase in cross-sectional area in going from the barrel bore to the chamber, the greater is the proportion of the shed rarefaction that is reflected as a compression. Therefore, the amount of the compression impulses sent back to the projectile is governed by the size of chamber diameter relative to the barrel bore diameter, $D/d*$ (see figure below). For the purposes of this qualitative description one may consider that with an infinite $D/d$ no part of a shed rarefaction would be transmitted as a rarefaction; all of it would reflect as a compression.
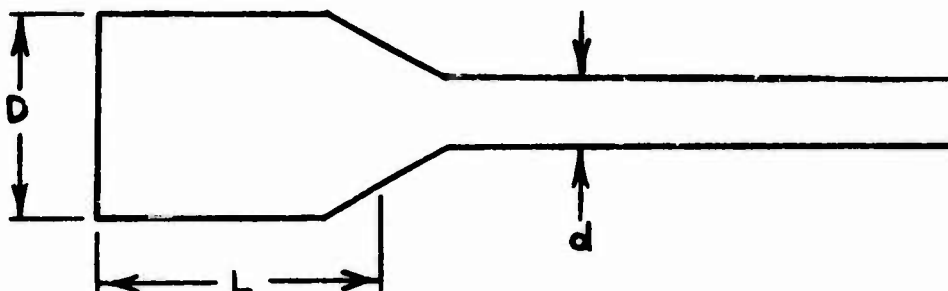


*The shape of the transition section between the chamber and the barrel is unimportant for this discussion.

The length from the change in area section to the breech, L, determines the time taken for rarefactions to reflect from the breech; the smaller L is, the more quickly the rarefactions reach the projectile and tend to lower the projectile velocity. Also, the smaller L, the more quickly the rarefactions travel back and forth in the chamber and lower the stagnation gas properties. This lowering, as noted above, decreases the effectiveness of reflected compressions.

It seems reasonable that there exists the possibility of a gun with a chambrage such that the reflected rarefaction and compression impulses sent forward to the projectile cancel one another, and consequently the projectile experiences no change of velocity as a result of the reflected impulses; the pressure behind the projectile would only drop from the accelerating projectile motion. This condition would simulate the constant area gun with the breech sufficiently removed from the projectile so that it doesn't influence the projectile's motion. (This is the case of the constant cross-sectional area gun with effectively infinite chamber volume.) The condition of complete cancellation of reflected impulses, however, could not persist indefinitely, because the compressions would become less effective as the projectile velocity increased and as the stagnation conditions in the chamber decreased. Hence, if the barrel were sufficiently long, the pressure of the gas behind the projectile would eventually decrease as a result of the reflected impulses, as well as from the accelerating projectile motion.

From this discussion it is evident that increasing either the chamber length L or the chamber diameter D (see figure above) will increase the projectile velocity. However, the two effects occur for different reasons. Increasing L prevents the rarefactions present from reaching the projectile before it is out of the barrel, and it increases the effectiveness of the compressions present. Increasing D decreases the amount of rarefactions present and increases the amount and effectiveness of the compressions present. Thus, increasing D provides the opportunity of increasing the projectile velocity to a greater value than by increasing L. This is seen from the following example. With infinite L and finite D equal to d, the projectile receives neither reflected rarefactions nor compressions. However, with infinite D and finite L, the projectile receives only compression impulses, and as a result the projectile velocity is greater than in the infinite L case.

It is apparent from the above reasoning that the method of treating chambrage in universal use, the replacing of the actual chamber by an equal volume imagined chamber of cross-sectional area equal to that of the bore, although qualitatively correct, may be quantitatively in error. Further, reasoning from this qualitative description of the effect of chambrage, one concludes that the projectile velocity is greater in a gun with chambrage than in an

equal chamber volume constant cross-sectional area gun for

(1) all cases in the early stages of projectile motion during which the compressions have reached the projectile and the reflected rarefactions have not, and

(2) in the later stages for the cases in which the effect of the compressions is larger than the effect of the reflected rarefactions*.

Hence, the conventional method of treatment yields muzzle velocities which are low in these cases. To determine whether the difference in velocity is significant or not requires a quantitative calculation. Below the results of such a calculation are given.

### III. QUALITATIVE DESCRIPTION OF THE BURNING PROPELLANT

In this discussion it is assumed that the propellant burns only in the chamber volume, not in the barrel section. (Although in reality the propellant may move during firing, it is to be noted that any increase in projectile velocity due to the movement of the propellant will be in the first approximation offset by the loss of energy of the propellant gas in moving the unburnt propellant. Corner, reference 1, points out that in the conventional gun the movement of the powder is unimportant.)

Since the burning of the propellant produces gas in the chamber, the burning can be viewed as the creation of compression impulses. Thus, from the burning propellant, compressions are issued which travel to the projectile. Because of the chambrage the compressions in going from the chamber to the barrel are partly reflected and the remaining portion transmitted at the decrease of area section. Hence, chambrage in this situation has the effect of delaying part of the compressions sent forward to the projectile by the burning propellant; however, these delayed compression impulses traveling back and forth in the chamber maintain or increase the chamber pressure, and therefore maintain or increase the effectiveness of the compressions sent to the projectile as described in the previous section.

If the pressure in the gun is limited to some peak value, the burning of the propellant can provide gas substantially at rest in the chamber at this given peak value. A less favorable burning rate would provide gas in the chamber at a lower pressure, and consequently the projectile velocity would be less. Therefore, the optimum burning rate is one which will provide gas at the peak stagnation pressure during the entire time that the projectile is in the barrel. The equivalence of this optimum burning rate condition to an optimum chambrage condition will be pointed out later.

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*If the gun were sufficiently long, the effect of the compressions would eventually become less than that of the rarefactions.

## IV.  QUALITATIVE DESCRIPTION OF WHAT OCCURS IN A GUN*

In an actual gun the ignition of the propellant causes the production of gas.  This gas increases the pressure in the chamber and behind the projectile by means of compressions, and the projectile begins to move.  During the early stages of the motion the gas directly behind the projectile rises higher and higher in pressure, primarily as a result of compressions it receives from the burning propellant in the chamber.  In addition compressions reflected at the change in area section are sent to the projectile.  At the same time and during the entire travel of the projectile in the barrel the pressure of the gas directly behind the projectile tends to drop (or drops) as a result of the accelerating projectile motion.  This pressure drop from the accelerating projectile motion is always present and increases with increasing projectile velocity.  However, in the initial stages the compressions, primarily from the burning gas and secondarily from the change in area section, greatly overcome the drop in pressure from the accelerating projectile motion, and the pressure behind the projectile continues to increase.

At some time after the projectile motion has begun, the rarefactions which have been reflected from the breech and partly transmitted through the change in area section reach the gas behind the projectile, tending to lower its pressure.  However, in the usual case the pressure behind the projectile continues to rise, although less rapidly, as a result of the compressions produced by the burning propellant until a peak pressure is reached.  Then, the pressure behind the projectile begins to decrease, primarily because the compressions from the burning propellant are weaker and are failing to maintain the peak stagnation pressure in the chamber.

After the propellant is completely burned, the gas receives weakening, i.e., less effective, reflected compressions from the change in area section.  (These compressions weaken as the chamber stagnation pressure decreases and as the projectile velocity increases.)  However, these compressions are insufficient to prevent the gas from dropping in pressure from the accelerating projectile motion and from the rarefactions reflected from the breech.  Therefore, the pressure behind the projectile drops in this final stage of projectile motion in the gun barrel.

In summary the tendencies for changes in pressure of the gas behind a projectile occurring in a gun and their causes are listed below.

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*In this section the effects of impulses which are sent to the projectile for the second time (as a consequence of successive reflections between breech or change in area section and projectile) are not discussed, since a description of the successive reflections is unessential and may confuse the qualitative picture presented.

(1) The drop in pressure from the accelerating projectile motion is present during the entire projectile travel and becomes greater with increasing projectile velocity.

(2) The rise in pressure from the compression impulses produced by the burning propellant is present during the burning period and is greatest for conventional guns in the middle of this period.

(3) The drop in pressure caused by the rarefactions reflected from the breech is present in the later stages of projectile motion.

(4) The rise in pressure from the compressions reflected from the change in area section is present during the entire projectile motion, but it is less in the later stages.

## V. APPROXIMATE METHODS OF ACCOUNTING FOR CHAMBRAGE IN GUNS

Since the exact solution for a gun with chambrage has not proved feasible, one can resort to several approximate methods of treating the problem. These would include the following:

(1) The change in area from the chamber to the barrel can be assumed to occur gradually. Then, the one dimensional characteristic method can be applied to this change in area section. The characteristic equations become for the change in area section

$$\frac{\partial}{\partial t}(u \pm \sigma) + (u \pm a)\frac{\partial}{\partial x}(u \pm \sigma) = \mp \frac{au}{A}\frac{dA}{dx}, \quad (1)$$

where u is the gas velocity, a is the sound velocity, $\sigma$ is the Riemann function (equal to $\int_o^p (dp/a\rho)_S$ ), and A is the cross-sectional area of the gas layer at position x and time t. (For the derivation and application of these equations, see references 2, 3 and 4.)

The usual one dimensional unsteady flow characteristic equations can be applied to the constant area barrel section and to the constant area chamber section. These are

$$\frac{\partial}{\partial t}(u \pm \sigma) + (u \pm a)\frac{\partial}{\partial x}(u \pm \sigma) = 0. \quad (2)$$

(2) It can be assumed that the change in area occurs suddenly from the larger diameter chamber to the smaller diameter barrel. At the discontinuous area position the change in state of the gas can be obtained by applying the steady state equations of continuity and energy:

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2 , \quad (3)$$
$$\frac{u_1^2}{2} + h_1 = \frac{u_2^2}{2} + h_2 ,$$

where subscript "1" refers to the chamber, and subscript "2" to the barrel, and where $\rho$ is the density, and $h$ the enthalpy of the gas.

The one dimensional characteristic equations (2) can be applied to the constant area chamber section and to the constant area barrel section.

(3) A third possible method of treatment is to assume that all the gas is at rest at the same thermodynamic state in the chamber  At the change in area section the steady flow energy equation can be applied.

$$ h_o(t) = \frac{u_2^2}{2} + h_2 , \qquad (4) $$

where $h_o(t)$ is the stagnation enthalpy in the chamber, and the sub.cript "2" refers to conditions at the entrance to the barrel.  It is to be noted that $h_o(t)$ here is a function of time and depends on the quantity of gas issuing from the chamber ($A_2 \rho_2 u_2$).  In the barrel section the one dimensional characteristic equations (2) can be applied.  This method of treatment is probably satisfactory for large chambered guns.

The determination of the accuracy of any of the above methods of treatment must await experimental investigation.  Such experimental investigation of chambrage is now being performed at the Naval Ordnance Laboratory and will be reported later.

For the purposes of ballistics the burning of the propellant must necessarily be accounted for.  It is advisable in the first two approximate treatments of chambrage above to treat the burning of the propellant in the chamber by the method of Carrière (reference 5); for the third approximate method the usual ballistic treatment of burning can to applied.  (See Corner reference 1.)

## VI.  THE CONDITIONS OF OPTIMUM CHAMBRAGE

It is proposed here to obtain quantitatively the effect of chambrage under the following conditions:

(a) The propellant burns instantaneously in a chamber of sufficient volume such that the gas in the chamber remains effectively at rest in a constant thermodynamic state at the peak pressure $p_o$ during the entire travel of the projectile in the barrel*.

(b) The chamber is physically shaped such that the high pressure gas effectively at rest in a constant thermodynamic state in the chamber is close to the beginning of the barrel section.

These two conditions taken together mean that the flow of gas from the chamber to the beginning of the barrel can be described by the steady flow energy equation.  This is demonstrated in the paragraph below.

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*Note that condition (a) does not mean that all of the gas in the chamber is at rest.

For unsteady gas flow without heat transfer the continuity equation and first law equation yield for a gas layer (see Appendix III)

$$\frac{D}{Dt}\left(\frac{u^2}{2} + h\right) = \frac{1}{\rho}\frac{\partial p}{\partial t} \quad , \tag{5}$$

where the left-hand side of the equation designates the time rate of change in $\frac{u^2}{2} + h$ for a particular layer of gas. This equation can be rewritten as

$$\Delta\left(\frac{u^2}{2} + h\right) = \int_0^{\bar{t}} \left(\frac{1}{\rho}\frac{\partial p}{\partial t}\right)dt \quad . \tag{6}$$

Here the left-hand member is the change in $\frac{u^2}{2} + h$ during the time taken for the gas layer to travel from the chamber to the beginning of the barrel. If this time $\bar{t}$ is small, by equation (6) the change in $\frac{u^2}{2} + h$ is small. Thus, if condition (b) is satisfied, the change in $\frac{u^2}{2} + h$ is negligible. If condition (a) is satisfied, the enthalpy for the gas at rest in the chamber remains constant. Therefore, the two conditions together result in the steady flow energy equation,

$$h_o = \frac{u_2^2}{2} + h_2 \quad , \tag{7}$$

where the subscript "o" refers to the stagnation state of the gas in the chamber, and subscript "2" refers to the state of the gas at the beginning of the barrel section. Here $h_o$ is constant and does not vary with time by condition (a).

By "optimum chambrage" it is meant that the two conditions (a) and (b) are satisfied and hence that equation (7) describes the relationship between the thermodynamic state of the gas at rest in the chamber and the gas at the beginning of the barrel section.

If either condition (a) or condition (b) is not satisfied, the projectile velocity will be less than if these conditions are satisfied. This conclusion is evident if one assumes that the gas is ideal and isentropically expands from rest from the chamber to the beginning of the barrel section. Then, equation (6) becomes

$$\frac{u_2^2}{2} = h_o\left[1 - \left(\frac{p_2}{p_o}\right)^{(\gamma-1)/\gamma}\right] + \int\frac{1}{\rho}\frac{\partial p}{\partial t}\,dt. \tag{8}$$

If condition (a) is not satisfied as a result of the stagnation pressure $p_o$ or enthalpy $h_o$ being lowered, or if condition (b) is not satisfied as a result of $\int\frac{1}{\rho}\frac{\partial p}{\partial t}\,dt$ being not negligible - note $\partial p/\partial t$ in chamber is always negative for guns when (a) is satisfied and (b) is not - then it is seen from equation (8) that the velocity $u_2$ of the gas at the beginning of the barrel section will be less for a given pressure $p_2$ at this point than if these two conditions are satisfied; thus, under these circumstances the projectile velocity (which is determined by the conditions at the beginning of the barrel) will be less.

Therefore, (a) and (b) are the conditions for maximum possible projectile velocity; under no circumstances can the projectile velocity

of a gun exceed this velocity*. Hence, the term "optimum chambrage" is rightfully applied to these conditions.

It seems logical that the optimum chambrage conditions can also be achieved by a propellant which burns in the chamber in such a manner as to provide gas at rest close to the beginning of the barrel section in a constant thermodynamics state at the maximum allowable pressure $p_o$ during the entire projectile travel. However, upon closer examination (in Appendix IV) it is found that if the propellant burns in a chamber whose diameter is less than about one and one-half of the barrel diameter, the conditions of optimum chambrage cannot be satisfied, and the projectile velocity will be somewhat less than the optimum chambrage projectile velocity. If the chamber diameter is larger than about one and one-half that of the barrel diameter, the "optimum burning propellant" can effectively achieve the conditions of optimum chambrage**.

Thus, in most instances the optimum chambrage conditions can be approached by either a large, well-shaped chamber or a propellant burning at the proper rate in a smaller chamber.

### VII. THE METHOD OF CALCULATION OF THE PROJECTILE VELOCITY FOR THE OPTIMUM CHAMBRAGE CONDITIONS

In order to determine quantitatively the effect of optimum chambrage on the muzzle velocity, a numerical calculation was made for a gun with optimum chambrage. The propellant gas is assumed to be ideal with a ratio of specific heats, $\gamma$, of 1.4. (The method used, however, can be applied to an imperfect gas or an ideal gas of any $\gamma$. See reference 2.) The gas in the chamber of the gun is

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*This is true if the propellant burns only in the chamber; it would not be true if the propellant burned while being carried with the projectile. In this case of a "rocket projectile" the mass to be accelerated is the projectile mass plus that of the propellant carried with the projectile. However, the pressure accelerating the projectile can be the maximum allowable gun barrel pressure.

**In a comparison between ballistic performance calculated from Pidduck's special solution and from the numerical equations of motion, Lorell (reference 6) uses equation (7) to correspond to the burning of liquid propellant in a constant cross-sectional area gun. He apparently employs a numerical characteristics technique similar to the one used here. The principal difference lies in the interpretation placed on the boundary condition (equation 7). As pointed out in the text, it is here maintained that the conditions of equation (7) cannot be achieved by a propellant burning in a constant cross-sectional area gun as Lorell assumes.

assumed to expand isentropically from its initial rest conditions to the conditions at the beginning of the constant cross-sectional area barrel. As the gun is one of optimum chambrage, equation (7) describes the expansion from the chamber to the beginning of the barrel. Insertion of the isentropic condition in this equation results in the following:

$$\frac{u_2^2}{2} = h_o - h_1 = \frac{a_o^2}{\gamma-1}\left[1 - \left(\frac{p_2}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right], (9)$$

where the subscript "2" refers to the state of the gas at the beginning of the barrel, and the subscript "o" refers to the constant state of the gas at rest in the chamber.

The assumptions of the theory used to describe the motion of the gas and projectile in the barrel section are the following:

(1) The gas motion is one dimensional in space.

(2) Each part of the gas expands isentropically.

(3) The projectile is unopposed by air pressure and frictional forces.

The problem is calculated for the case in which the back end of the projectile is initially positioned at the beginning of the uniform area barrel section. The following dimensionless variables are employed in the solution:

$$\bar{x} = \frac{p_o A x}{M[2/(\gamma-1)]^2 a_o^2} = \frac{p_o A x}{25 a_o^2 M} ,$$

$$\bar{t} = \frac{p_o A t}{M[2/(\gamma-1)] a_o} = \frac{p_o A t}{5 a_o M} ,$$

$$\bar{u} = \frac{u}{[2/(\gamma-1)] a_o} = \frac{u}{5 a_o} , \qquad (10)$$

$$\bar{a} = \frac{a}{[2/(\gamma-1)] a_o} = \frac{a}{5 a_o} ,$$

$$\bar{p} = \frac{p}{p_o} , \qquad \bar{\sigma} = \frac{\sigma}{\sigma_o} = \frac{\sigma}{2 a_o/(\gamma-1)} = \frac{\sigma}{5 a_o} .$$

The symbols employed in the above equations are defined in Table II at the end of this report.

Since the gas is ideal and each part expands isentropically,

$$\bar{\sigma} = \frac{2}{\gamma-1}\,\bar{a} = 5\,\bar{a}\quad,$$

$$\bar{p} = \bar{\sigma}^{\,2\gamma/(\gamma-1)} = \bar{\sigma}^{\,7}.$$

(11)

Equation (9) becomes

$$\bar{u}_2^2 = \frac{1 - \bar{\sigma}_2^2}{2/(\gamma-1)} = \frac{1 - \bar{\sigma}_2^2}{5}\quad.$$

(12)

Thus, the gas thermodynamic state at the beginning of the barrel section is related to the constant state of the gas at rest in the chamber section by equation (12).

The one dimensional unsteady characteristic equations (2) become for the gas in the barrel

$$\frac{\partial}{\partial t}(\bar{u} \pm \bar{\sigma}) + (\bar{u} \pm \bar{a})\frac{\partial}{\partial x}(\bar{u} \pm \bar{\sigma}) = 0.$$

(13)

The equation for the projectile acceleration can be written in terms of the pressure of the gas directly behind the projectile.

$$\frac{d\bar{u}}{dt} = \bar{p}.$$

(14)

From the boundary conditions (12) and (14) and the gas characteristic equations (13) the entire behavior of gas and projectile during the expansion process can be obtained.

To calculate the projectile velocity from equations (12), (13) and (14) it is necessary to use the step-by-step numerical characteristics method. (This method is outlined in references 2, 3, 7 and in many other reports.) This requires that a characteristic net (two sets of intersecting characteristic lines: one with slope $\bar{u}+\bar{a}$ along which $\bar{u}+\bar{\sigma}$ is constant, another with slope $\bar{u}-\bar{a}$ along which $\bar{u}-\bar{\sigma}$ is constant) be used. The characteristic lines forming the net can be interpreted as the paths of disturbance impulses, since as one goes along a characteristic line, one travels at the same speed as a disturbance would, that is, at the local velocity of sound relative to the moving gas.

A schematic drawing of the characteristics diagram in the position plane used in this calculation is shown in Figure 1, while a portion of the actual characteristics diagram is shown in Figure 2. In Figure 1 the points O, A, B, C, D, etc., are points on the projectile path; the line, $\bar{X} = 0$, represents the beginning of the constant area barrel.

To begin the numerical solution, point A on the projectile path was taken with a velocity $\bar{u}$ equal to a small value, .01. The pressure $p_0$ at this point was assumed to be equal to 1.0. (Thus, the assumption

was made that the velocity which the projectile had reached was suffi-
ciently small that the pressure had not significantly changed at this
stage.)  From points O and A, point 1 was calculated; from 1 and A,
point B was calculated, and so on.  In order to check the assumption
that the pressure had not dropped significantly at the point $\bar{u} = .01$,
the calculation was redone for a short way by starting with a point
with $\bar{u}$ set at .009 and $\bar{p}$ at 1.0.  The recalculated projectile path was
compared to that of the first calculation.  The two paths agreed within
the accuracy of the calculation; thus, the assumption used was justified.

Whether the characteristic net was becoming too coarse or not was
determined at various times by interpolating between two calculated points
and then by using the interpolated point and one of the calculated points
to recalculate the other calculated point.  If the two sets of values
for this one point agreed within the accuracy of the calculation, the
net was considered to be sufficiently fine.  To prevent the net from
becoming too coarse a parabola was fitted through the points (in Figure
1) R, S and AA.  With this parabola the points T, U, V, W, X, Y and Z
were calculated and used to continue the characteristics net.

### VIII.  THE CALCULATED PROJECTILE VELOCITY FOR A GUN WITH OPTIMUM CHAMBRAGE

The calculated values for points along the projectile path are
given in Table I.  The accuracy of these values is about 1/10%.

To determine the maximum projectile velocity for the optimum cham-
brage gun, the impulses traveling downstream toward the projectile may
be examined.  For each of these impulses the quantity $\bar{u} + \bar{\sigma}$ is a
constant, a different constant for each impulse.  This sum of $\bar{u} + \bar{\sigma}$
can be obtained in terms of the velocity at the beginning of the barrel
from equation (12);

$$\bar{u}_2 + \bar{\sigma}_2 = \bar{u}_2 + \sqrt{1 - 2\bar{u}_2^2/(\gamma-1)} = \bar{u}_2 + \sqrt{1 - 5\bar{u}_2^2} \quad (15)$$

It is evident from equation (15) that at the beginning of the projectile
motion the sum of $\bar{u}_2 + \bar{\sigma}_2$ is 1.0; as the gas velocity at the beginning of
the barrel section increases, the sum of $\bar{u}_2 + \bar{\sigma}_2$ increases.  The maximum
velocity with which gas can issue from the chamber into the barrel is
the local velocity of sound*; this velocity is from the equation (12)

$$u_{2 (MAX)} = \sqrt{\frac{2a_0}{\gamma + 1}} \quad ,$$

$$\bar{u}_{2 (MAX)} = \frac{\gamma - 1}{\sqrt{2(\gamma+1)}} \quad . \qquad (16)$$

The sum of $\bar{u}_2 + \bar{\sigma}_2$ is a maximum when the gas velocity is this value;
hence, from (15) and (16)

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*This is true for the unsteady gas flow case dealt with here, as well
as for the usual steady flow case.

$$\left(\overline{u}_2 + \overline{\sigma}_2\right)_{(MAX)} = \sqrt{\frac{\gamma+1}{2}} \quad . \tag{17}$$

The calculated velocity values at the beginning of the barrel section - these values are not given here - demonstrate that the maximum gas velocity of equation (16) is approached very rapidly, but is only actually reached after an infinite period of time. Thus, during the firing of an optimum chambrage gun the sum of $\overline{u} + \overline{\sigma}$ of the downstream impulses and hence, of the gas directly behind the projectile which these impulses reach, increases rapidly from the value 1 to close to $\sqrt{(\gamma+1)/2}$ . Therefore, the maximum projectile velocity, which occurs when the Riemann function, $\overline{\sigma}$, and the pressure of the gas behind the projectile become zero, is

$$\overline{u}_{\text{projectile}\,(MAX)} = \left(\overline{u} + \overline{\sigma}\right)_{(MAX)} = \sqrt{\frac{\gamma+1}{2}} , \tag{18}$$

which for $\gamma = 1.4$ is equal to 1.095445 or dimensionally, 5.477225 $a_o$.

The calculated values of Table I show that the quantity $\overline{u} + \overline{\sigma}$ for the gas directly behind the projectile can be taken equal to about 1.096 above a projectile velocity of .2. Below this velocity, $\overline{u} + \overline{\sigma}$ can be approximated as a straight line function of $\overline{u}$. These considerations yield the following approximate equations for the pressure behind the projectile in an optimum chambrage gun in terms of the projectile velocity:

$$\overline{p} = \left(1 - .52\,\overline{u}\right)^7 \quad \text{for} \quad 0 \leq \overline{u} \leq .2^*,$$

$$\tag{19}$$

$$\overline{p} = \left(1.096 - \overline{u}\right)^7 \quad \text{for} \quad .2 \leq \overline{u} \leq 1.096 ,$$

or in dimensional symbols

$$p = p_o\left(1 - \frac{u - .48u}{5\,a_o}\right)^7 \quad \text{for} \quad 0 \leq u \leq a_o^*,$$

$$\tag{19a}$$

$$p = p_o\left(1 - \frac{u - .48a_o}{5\,a_o}\right)^7 \quad \text{for} \quad a_o \leq u \leq 5.48\,a_o .$$

Similar expressions could be written by analogy for an ideal gas of $\gamma$ not equal to 1.4. For example, at higher velocities the pressure would be approximately

$$\overline{p} = \left(\sqrt{\frac{1+\gamma}{2}} - \overline{u}\right)^{2\gamma/(\gamma-1)} . \tag{20}$$

*For the region $0 \leq u \leq .06$ the steady flow equation, $\overline{p} = \overline{(1-5\overline{u}^2)}^{3.5}$, is a better approximation.

It is informative to compare the optimum chambrage gun with a constant cross-sectional area gun with infinite chamber volume. While the latter gun is one in which the projectile receives neither reflected compressions nor rarefactions, the optimum chambrage gun is one in which the projectile receives only reflected compressions. Therefore, the difference between the two guns is a measure of the compressive effect of chambrage. (It is to be noted that these two guns would be equivalent from the standpoint of the ordinary ballistic treatment of chambrage.)

The equations for the gas properties directly behind the projectile in a constant cross-sectional area infinite chamber volume gun are (see references 2 and 7)

$$\bar{u} + \bar{\sigma} = 1,$$

$$\bar{p} = (1 - \bar{u})^{2\gamma/(\gamma-1)},$$

$$\bar{x} = \frac{(\gamma-1)^2}{2(\gamma+1)}\left[\frac{\frac{2}{\gamma-1} - \frac{\gamma+1}{\gamma-1}(1-\bar{u})}{(1-\bar{u})^{(\gamma+1)/(\gamma-1)}} + 1\right]. \tag{21}$$

A comparison between the two guns is given in Figures 3 and 4, where plots of dimensionless projectile velocity versus dimensionless projectile travel are shown for a $\gamma = 1.4$ propellant gas. It is seen that, as expected, the velocity of the optimum chambrage gun is always greater than that of the infinite chamber volume constant area gun for equal gun lengths (i.e., for equal projectile travel). The difference in velocity between the two guns becomes larger for increasing gun lengths until the optimum chambrage projectile velocity is about 2 1/4 times the initial sound velocity (at which time it is 28% higher than the constant area gun velocity). Thereafter, the difference in projectile velocity between the two guns remains about the same value, slightly under one-half of the initial sound velocity*. Figure 5 is a comparison between the two different guns of the pressure behind the projectile for varying projectile velocities. Here, the increase in pressure as a result of the compressions is quite evident.

The $\bar{u}$ vs. $\bar{x}$ comparison for the two guns for $\gamma = 1.25$, an average value for gun propellants, is shown in Figure 6. This plot was obtained by analogy to the $\gamma = 1.4$ results - see equation (20) - as outlined in Appendix V. It is to be emphasized that in an actual gun the projectile velocity for a given projectile travel (and the pressure behind the projectile for a given projectile velocity) cannot exceed the values of the optimum chambrage gun as shown in Figures 3, 4, 5 and 6. (For this consideration $p_o$ and $a_o$ are the values of the gas at rest in the chamber of the actual gun at a time previous to that considered, when the gas is at a maximum pressure.)

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*This conclusion can be arrived at from equations (19) and (21).

In conclusion it can be stated that this calculation demonstrates that chambrage can have an important effect on the muzzle velocity of guns. It is recommended that in order to calculate approximately the chambrage effect for a _particular_ gun the methods of section $V$ be used.

## TABLE I

### POINTS ON PROJECTILE PATH

| POINT | $\bar{x}$ | $\bar{t}$ | $\bar{u}$ | $\bar{\sigma}$ | $\bar{p}$ | $\bar{a}$ |
|-------|-----------|-----------|-----------|----------------|-----------|-----------|
| O | .00000 | .00 | .00 | 1.0 | 1.0 | .20 |
| A | .00005 | .01 | .01 | 1.0 | 1.0 | .20 |
| B | .0000554067 | .0105268 | .0105250 | .998998 | .993010 | .199800 |
| C | .0000617411 | .0111125 | .0111081 | .998720 | .998529 | .199958 |
| D | .0000692253 | .0117670 | .0117598 | .998952 | .992686 | .199790 |
| E | .0000781588 | .0125037 | .0124927 | .999544 | .996810 | .199909 |
| F | .0000889376 | .0133388 | .0133230 | .998838 | .991893 | .199768 |
| G | .000102112 | .0142936 | .0142714 | .999234 | .994651 | .199847 |
| H | .000118443 | .0153957 | .0153652 | .998619 | .990373 | .199724 |
| I | .000139032 | .0166823 | .0166403 | .998810 | .991698 | .199762 |
| J | .000165492 | .0182036 | .0181459 | .998222 | .987621 | .199644 |
| K | .000200299 | .0200309 | .0199502 | .998164 | .987219 | .199633 |
| L | .000247359 | .0222665 | .0221520 | .997488 | .982547 | .199498 |
| M | .000313198 | .0250652 | .0248976 | .997049 | .979525 | .199410 |
| N | .000409302 | .0286704 | .0284162 | .996013 | .972424 | .199203 |
| O | .000557562 | .0334918 | .0330843 | .994774 | .963988 | .198955 |
| P | .000803863 | .0402720 | .0395691 | .992534 | .948893 | .198507 |
| Q | .00125853 | .0505197 | .0491670 | .988813 | .924272 | .197763 |
| R | .00224508 | .0678384 | .0647617 | .981368 | .876644 | .196274 |
| S | .00508249 | .103535 | .0942130 | .963968 | .773459 | .192794 |
| T | .00754581 | .128144 | .112154 | .951994 | .708663 | .190399 |
| U | .0105371 | .152111 | .128630 | .939520 | .646164 | .187904 |
| V | .0149684 | .186006 | .149314 | .923913 | .574669 | .184783 |
| W | .0154755 | .189404 | .151325 | .922268 | .567545 | .184454 |
| X | .0175828 | .202997 | .159217 | .915687 | .539796 | .183137 |
| Y | .0189640 | .211493 | .164031 | .911576 | .523058 | .182315 |
| Z | .0196729 | .215740 | .166404 | .909523 | .514868 | .181905 |
| AA | .0203944 | .219988 | .168758 | .907470 | .506787 | .181494 |
| BB | .0505542 | .371283 | .231100 | .853025 | .328649 | .170605 |
| CC | .128655 | .662417 | .305429 | .783946 | .181971 | .156789 |
| DD | .688406 | 2.114437 | .465566 | .628181 | .0386006 | .125636 |
| EE | .898316 | 2.557810 | .481310 | .612713 | .0324190 | .122543 |
| FF | 2.847078 | 6.286648 | .563927 | .530940 | .0118937 | .106188 |
| GG | 7.818634 | 14.598786 | .632288 | .462907 | .0045546 | .0925814 |
| HH | 15.719393 | 26.694660 | .674068 | .421241 | .0023535 | .0842482 |
| II | 40.219361 | 61.587443 | .730231 | .365156 | .0006656 | .0730312 |

## TABLE II

### NOMENCLATURE

$a$   - Velocity of sound

$A$   - Cross-sectional area of gun barrel

$d$   - Diameter of gun barrel

$D$   - Diameter of gun chamber

$h$   - Enthalpy

$L$   - Distance from breech to change in area section

$M$   - Mass of projectile except in Appendix IV where M designates Mach number

$p$   - Pressure

$t$   - Time

$u$   - Mass gas velocity

$U$   - Internal energy of gas

$X$   - Position of gas element from beginning of barrel

$\gamma$   - Specific heat ratio

$\mu$   - Viscosity coefficient

$\rho$   - Gas density

$\sigma$   - Riemann function, defined as $\displaystyle\int_{0}^{p} (dp/a\rho)_s$   $s \equiv$ entropy

A symbol with a bar is dimensionless and is related to the dimensional quantities by equations (10). The subscript "o" refers to the constant state of the gas at rest in the chamber; the subscript "2" refers to the gas at the beginning of the barrel section.

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## APPENDIX I

### THE PRESSURE DROP FROM THE ACCELERATING PROJECTILE MOTION

In order to study the pressure drop from the accelerating projectile motion the special case in which the gas directly behind the projectile receives no reflected impulses should be examined; in this case the pressure behind the projectile drops only as a result of the accelerating projectile. Such a case is a constant cross-sectional area gun with an infinite chamber volume.

From the characteristic equations (2) of the text it can be shown for the gas in an infinite chamber constant cross-sectional area gun that (see reference 2)

$$u + \sigma = \sigma_o ,$$

$$\text{or} \quad du = -d\sigma ,$$
(a)

$$\text{or} \quad \rho a \, du = -dp .$$
(b)

In particular equation (b) expresses the differential drop in pressure of the gas directly behind the projectile in terms of the gas density, sound velocity, and differential velocity increase of the projectile. It is the analytic expression for the pressure drop from the accelerating projectile motion. For an ideal gas isentropically expanding, equation (b) can be integrated to give

$$p = p_o \left[ 1 - \frac{(\gamma-1) \, u}{2 \, a_o} \right]^{2\gamma/(\gamma-1)}$$
(c)

It is seen from this equation that the pressure drop becomes larger for larger projectile velocities, and that if the projectile ceases to accelerate (i.e., the velocity is constant), there will be no additional drop in pressure.

Further insight into the pressure drop from the accelerating projectile motion can be gained by examination of equation (b). From this equation it is evident that the pressure drop for a given velocity increase is determined only by the value of $\rho a$ of the gas; this quantity is known as the "characteristic acoustic resistance" of the gas. It is the fundamental propellant gas property determining the drop in pressure as a result of the accelerating projectile motion. Therefore, for high muzzle velocities one would desire a propellant gas with a small characteristic acoustic impedance as a function of pressure along the gas isentrope.

APPENDIX II

## THE PRESSURE RISE OF THE GAS BEHIND THE PROJECTILE AS A
## RESULT OF COMPRESSION IMPULSES

The effect of the compression impulses on the gas directly behind the projectile can be studied by examining the case when a projectile in an optimum chambrage gun is made to accelerate instantaneously to a given velocity and thereafter is made to move with this constant velocity. In this case the projectile sheds rarefactions only at the first instant; after this it sheds no rarefactions, because its velocity is constant. Therefore, the pressure behind the projectile after the first instant will rise only as a result of the reflections from the change in area section; since this change in area section is the optimum chambrage configuration, the pressure rise will be a maximum for the given projectile velocity.

If the optimum chambrage condition of equation (12) of the text,

$$\frac{2}{\gamma-1} \bar{u}_2^2 = 1 - \bar{\sigma}_2^2$$

is applied to the beginning of the barrel, and the characteristics equation (2) of the text is applied to the constant area barrel, the pressure of the gas behind the projectile in this case of constant projectile velocity can be easily obtained.

The rarefaction impulses which are shed by the projectile in this first instant, travel to the change in area section, where they are reflected as compression impulses. These compression impulses upon reaching the gas behind the projectile raise the pressure of this gas; they are reflected as compression impulses from the projectile and begin to travel back to the change in area section. If the projectile velocity is too large, these compression impulses will not reach the change in area section; if the projectile velocity is not too large, these impulses will reach the change in area section and be reflected as rarefactions. When these rarefactions reach the projectile, they lower the pressure of the gas behind the projectile, are reflected, and this reflection process continues, resulting in a decreasing amplitude rise and fall of the pressure of the gas behind the projectile.

Thus, it is that as the velocity of the projectile rises instantaneously to the constant value, $\bar{u}_p$, the pressure of the gas behind the projectile drops to the value,

$$\bar{p} = \left(1 - \bar{u}_p\right)^{2\gamma/(\gamma-1)} .$$

Then, this pressure will rise to some maximum value as a result of the reflected compression impulses and (a) remain at this pressure in the high velocity cases, or (b) will oscillate about a lower final pressure in the low velocity cases. This behavior is shown in Figure 7, which

is a plot of the pressure behind the projectile as a function of the constant projectile velocity for a $\gamma = 1.4$ gas. Shown is the pressure after the initial instant, the maximum pressure, and the final pressure. It is rather astonishing that for dimensionless velocities less than about .085 the pressure behind the projectile rises to a value <u>above</u> the initial value of the gas pressure.

(Since there are compressions present in the gas, there is a possibility that compression shocks occur. It seems probable that such shocks will occur in the high velocity cases; these would alter the final pressure, but probably not the maximum pressure behind the projectile. The existence of shocks has not been accounted for in Figure 7 and would not change the conclusions of this appendix.)

Figure 7 demonstrates that for a given constant projectile velocity there corresponds a maximum rise in pressure possible by the action of the compression forces. By comparison with Figure 5 it is seen that the pressure rise in the constant velocity case is always larger than that which occurs in an optimum chambrage gun in which the projectile velocity increases as a result of the gas pressure behind it. Further, it is seen from Figure 7 that as the projectile velocity becomes larger (after $\bar{U}_p = .085$) the maximum pressure attainable by the gas as a result of the compressions becomes less.

Finally, Figure 7 demonstrates that the pressure rise behind the projectile is dependent also on the thermodynamic state of the gas in the chamber at rest. This is seen by noting from the figure that if either $p_e$ or $a_e$ (and consequently $h_e$ for this ideal gas) drops, the rise in pressure for a given projectile velocity is less.

## APPENDIX III

DERIVATION OF $\frac{D}{Dt}\left(\frac{u^2}{2} + h\right) = \frac{1}{\rho}\frac{\partial p}{\partial t}$

If the first law of thermodynamics is applied to a gas element, the following equation is obtained (Goldstein, reference 5, page 603, et. seq.):

$$\rho\frac{DU}{Dt} = -p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + k\nabla^2 T + \Phi , \quad \text{(a)}$$

where the left side represents the product of the gas density by the time rate of change of the internal energy of the gas element; u, v and w are the velocity components in the x, y and z directions; $k\nabla^2 T$ is the rate of heat conduction to the gas element (T being the temperature), and the dissipation function $\Phi$ is the rate of work done by the fluid element against viscosity.

The equation of continuity is

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right). \quad \text{(b)}$$

Inserting (b) into (a), one obtains

$$\rho\frac{DU}{Dt} = \frac{p}{\rho}\frac{D\rho}{Dt} + k\nabla^2 T + \Phi . \quad \text{(c)}$$

The equation of motion for the gas element in the x direction is

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu\left[\nabla^2 u + \frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right]; \quad \text{(d)}$$

two similar equations could be written for the y and z directions. The second term on the right-hand side of equation (d) is the viscous stress component on the element in the x direction and shall be designated as $\psi_x$. The equations of motion can thus be written as

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \psi_x ,$$
$$\rho\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \psi_y , \quad \text{(e)}$$
$$\rho\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \psi_z .$$

If the three equations (e) are multiplied by u, v and w respectively and added to equation (c), the following expression is obtained:

$$\rho\frac{D}{Dt}\left[U + \frac{1}{2}(u^2 + v^2 + w^2)\right] = \frac{p}{\rho}\frac{D\rho}{Dt} + k\nabla^2 T$$
$$\quad \text{(f)}$$
$$+ \Phi + u\left(-\frac{\partial p}{\partial x} + \psi_x\right) + v\left(-\frac{\partial p}{\partial y} + \psi_y\right) + w\left(-\frac{\partial p}{\partial z} + \psi_z\right).$$

Adding $\rho \dfrac{D^{p/\rho}}{Dt}$ to each side of the above equation results in

$$\rho \frac{D}{Dt}\left[h + \tfrac{1}{2}(u^2 + v^2 + w^2)\right] = \frac{\partial p}{\partial t} + K\nabla^2 T \tag{g}$$

$$+ \Phi + (u\psi_x + v\psi_y + w\psi_z),$$

where the enthalpy h has been inserted for $U + \frac{p}{\rho}$. The left-hand side of this equation represents the density multiplied by the time rate of change of the sum of the enthalpy and the kinetic energy of the gas element.

If the gas flow is essentially one dimensional in the x direction, the heats conducted between gas elements within layers normal to the x direction are equal and opposite, as are the viscous effects, and their sums are zero. If also the viscous effects and heats transferred in the x direction are negligible, (as they are in all guns), equation (g) becomes for a gas layer*

$$\rho \frac{D}{Dt}\left(h + \frac{u^2}{2}\right) = \left(\frac{\partial p}{\partial t}\right)_x, \tag{h}$$

which is equation (5) of the text. Thus, for flows which are one dimensional (or approximately so, in which case an integrated average over the gas layer can be employed) with the heat and viscous effects in the direction of flow negligible, the equation (h) obtains for the uniform gas layer.

For the reversible adiabatic (i.e., isentropic) flow of each gas element there is no heat conducted to the element, and there are no viscous effects. Thus, for each element

$$\frac{D}{Dt}\left[h + \tfrac{1}{2}(u^2 + v^2 + w^2)\right] = \frac{1}{\rho}\frac{\partial p}{\partial t}. \tag{i}$$

If further the conditions at two sections in the flow (e.g., at the entrance and the exit to the change in area section of a chambered gun) are one dimensional in space, between these two sections

$$\Delta\left(h + \frac{u^2}{2}\right) = \int \frac{1}{\rho}\frac{\partial p}{\partial t}\, dt$$

for each gas element. Thus, again for an isentropic flow the equation (5) of the text is applicable, even though the flow is not one dimensional.

In general, flows are not one dimensional, and the effects of the gradients in the direction of the flow (i.e., the x direction) may not be negligible. For such flows in which gas expands from a uniform

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*It is understood that no heat is added to the gas through the walls.

one dimensional region (as near the breech of a gun), the value of the integrated x component of velocity in the y-z gas layer would be less than in the corresponding one dimensional flow case, for these two reasons:

(1) Because of the existence of velocity components in the three directions some of the kinetic energy of the gas would be in the form of $\frac{1}{2}(v^2 + w^2)$, leaving less in the form of $\frac{1}{2} u^2$.

(2) Although the heats transferred between gas elements within gas layers normal to the x direction still tend to cancel each other, as do the viscous effects, in this non-uniform expanding gas flow, these effects in the x direction of flow decrease the gradients which exist within the gas; thus, the slow moving gas would by viscous action slow up the fast moving gas ahead of it.

Hence, the arguments of section VI, that optimum chambrage yields a maximum velocity, is valid in the non-uniform general flow case.

APPENDIX IV

## THE CONDITIONS UNDER WHICH THE OPTIMUM BURNING RATE IS
## EQUIVALENT TO OPTIMUM CHAMBRAGE

The case is examined of a propellant at the breech of a chambered
gun burning so that the propellant gas at rest maintains a maximum
allowable pressure $p_e'$ in the chamber; this gas at rest is assumed to
be physically close to the change in area section. The conditions at
the beginning of the barrel for such a gun (an optimum burning rate gun)
will be compared to corresponding conditions for an optimum chambrage
gun with gas at rest in the chamber at the pressure $p_e'$.

With the assumption of one dimensional flow the optimum condition
for the expanding gas (the condition yielding the maximum gas velocity
at the beginning of the barrel section) would be described by the steady
flow energy equations as in the case of optimum chambrage. Thus, the
steady flow energy equations describing the flow for the optimum burning
rate gun are

$$Q + h_o'^* = h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} = h_o, \quad (a)$$

where Q is the heat of reaction, the subscript "o" refers to the high
pressure conditions at the back of the burning (see sketch below) zone,
subscript "1" refers to conditions in the chamber after burning,
subscript "2" refers to conditions at the beginning of the barrel,
and subscript "o" refers to the stagnation condition for the flow
at "1".
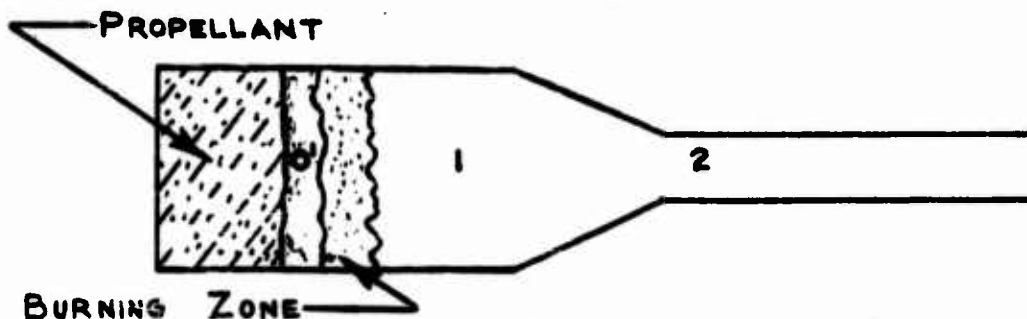


Equation (a) demonstrates that the burning can be visualized as
an addition of heat to a propellant which is initially at the rest state
"o" at low temperature and high pressure $p_e'$. (It is to be noted that
the burning propellant produces high temperature gas at "1" that is
not at rest. The gas at rest at the maximum allowable pressure $p_e'$ is
at the back of the burning zone.) Further, the equation shows that
the gas at "2" can be considered to have expanded from the stagnation
state with enthalpy $h_o$, adiabatically.

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*The burning of the propellant is viewed here as occurring instanta-
neously; thus, the steady flow energy equation is used.

If the flow is considered as an isentropic ideal gas flow, the applicable steady flow diabatic and adiabatic equations (see, for example, Hall, reference 9) yield the following expressions:

$$\frac{p_o}{p_o'} = \frac{1 + \gamma M_1^2}{\left(1 - \frac{\gamma-1}{2} M_1^2\right)^{\gamma/(\gamma-1)}} \quad , \quad \text{(b)}$$

$$\frac{A_1}{A_2} = \frac{M_2}{M_1}\left(\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}\right) \quad , \quad \text{(c)}$$

where M is the Mach number, $\frac{u}{a}$.

It is seen from the above equations that when the ratio of the chamber area to that of the barrel, $A_1/A_2$, is infinite, $M_1$ is zero, and $p_o$ is equal to $p_o'$. For this case the optimum burning rate gun is equivalent to the optimum chambrage gun, since both expansions can be considered to occur adiabatically from the stagnation state "O" at the same enthalpy and pressure. However, when the ratio $A_1/A_2$ is finite, $M_1$ is greater than zero, and $p_o'$ is greater than $p_o$. Therefore, in this latter case if the optimum burning rate gun is limited to the same peak pressure as the optimum chambrage gun, the stagnation pressure of the propellant gas after burning (at point "1") for the optimum burning rate gun ($p_o$) is lower than the stagnation pressure of the propellant gas in the optimum chambrage gun after burning (equal to $p_o'$). Consequently, since both gases expand from the same stagnation enthalpy, the velocity at the beginning of the barrel in the optimum burning rate gun with finite $A_1/A_2$ will be less than that of the optimum chambrage gun. How much less depends on the ratio of $p_o'/p_o$.

The ratio $p_o'/p_o$ has been calculated from equations (b) and (c) as a function of $A_1/A_2$ for the case where $M_2$ is equal to one (its maximum value). For $A_1/A_2$ equal to one,

$$\frac{p_o'}{p_o} = (\gamma + 1)\left(\frac{2}{\gamma-1}\right)^{\gamma/(\gamma-1)}$$

$$\quad \text{(d)}$$

$$= 1.269 \quad \text{for} \quad \gamma = 1.4 .$$

A listing is made below of $p_o'/p_o$ for various values of $A_1/A_2$ for $\gamma = 1.4$ at $M_2=1$.

| $\frac{A_1}{A_2} \rightarrow$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\frac{p_o'}{p_o} \rightarrow$ | 1.27 | 1.06 | 1.03 | 1.015 |

Since the condition $M_2=1$ was taken, the above values are the maximum values of the ratio $p_a'/p_a$   for the given area ratio $A_1/A_2$  ; for any lower value of $M_2$, the values of $p_a'/p_a$ are less.

From this discussion it is apparent that the optimum burning rate gun is equivalent to the optimum chambrage gun only when $A_1/A_2$ is greater than about 2 or 3.  If $A_1/A_2$ is less than 2 or 3, the projectile velocity for the optimum burning rate gun will be less than that of the optimum chambrage gun.  Further, by the use of equations (b) and (c) the calculations made for an optimum chambrage gun can be changed to apply to an optimum burning rate gun of finite $A_1/A_2$.

## APPENDIX V

### METHOD OF OBTAINING THE APPROXIMATE $\bar{u}$ VS $\bar{X}$ CURVE FOR AN OPTIMUM CHAMBRAGE GUN WITH $\gamma$ EQUAL TO 1.25

By analogy with the $\gamma = 1.4$ calculation the projectile velocity versus the projectile travel for a $\gamma = 1.25$ gas can be obtained. For the region $0 \le \bar{u} \le .08$ the steady flow energy equation was found to be a good approximation to the optimum chambrage case for the $\gamma = 1.4$ gas, although this approximation yielded larger velocities for given travels. For the region $.08 < \bar{u} \le .2$ a good approximation for $\bar{u} + \bar{\sigma}$ was found to be a straight line function of $\bar{u}$. Above $\bar{u} = .2$ the quantity $\bar{u} + \bar{\sigma}$ could be approximately taken as equal to the limiting velocity for the $\gamma = 1.4$ case, although this too gave higher velocities for given travels.

These considerations lead by analogy to the following equations for $\bar{X}$ as a function of $\bar{u}$ for the $\gamma = 1.25$ gas in an optimum chambrage gun:

$$\bar{X} = \frac{1}{64}\left[\frac{\bar{u}}{(1 - 5\bar{u}^2)^4}\right] \qquad \text{for} \quad 0 \le \bar{u} \le .08 ,$$

$$\bar{X} = \bar{X}_{.08} + \frac{1}{72}\left[\frac{8 - 9(1 - .7\bar{u})}{(1 - .7\bar{u})^9}\right]_{\bar{u}=.08}^{\bar{u}} \quad \text{for} \quad .08 < \bar{u} \le .2 ,$$

$$\bar{X} = \bar{X}_{.2} + \frac{1}{72}\left[\frac{8 - 9(\sqrt{1.125} - \bar{u})}{(\sqrt{1.125} - \bar{u})^9}\right]_{\bar{u}=.2}^{\bar{u}} \quad \text{for} \quad .2 \le \bar{u} \le \sqrt{1.125} .$$

These approximate equations will yield slightly higher velocities than an exact calculation would yield for the given projectile travel. A plot of $\bar{u}$ vs $\bar{X}$ as calculated from these equations is given in Figure 6.

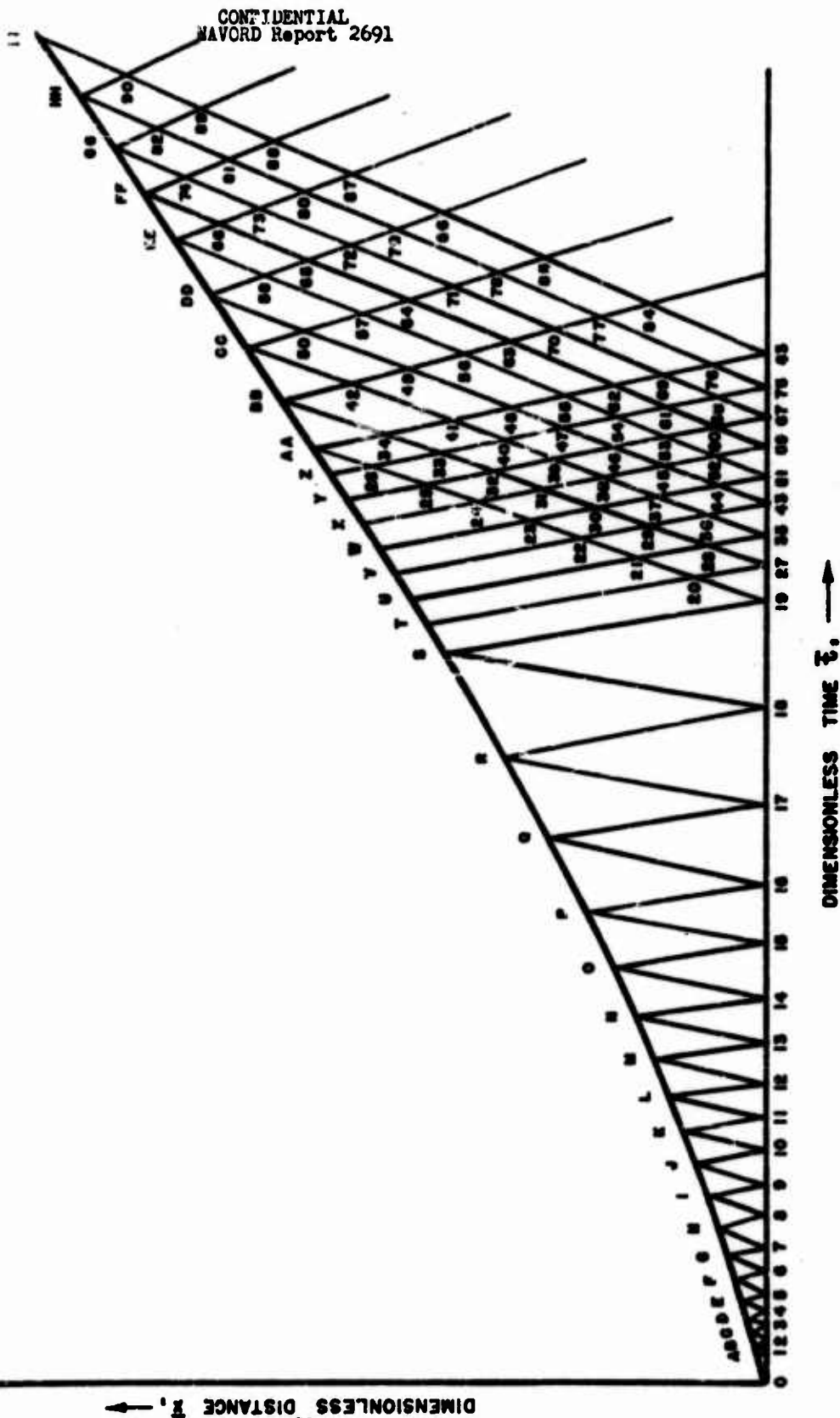FIG. I SCHEMATIC CHARACTERISTICS DIAGRAM FOR OPTIMUM CHAMBRAGE GUN

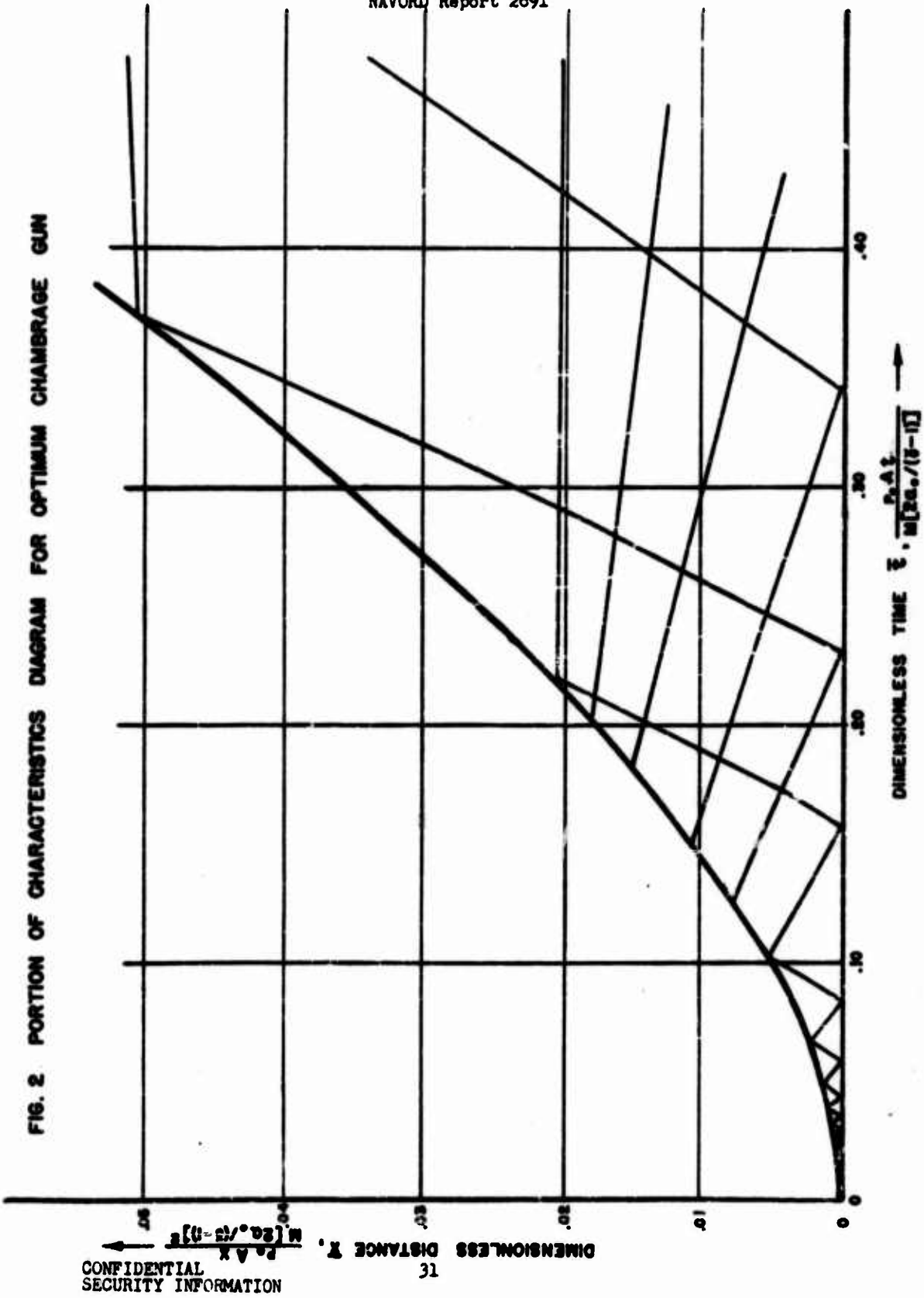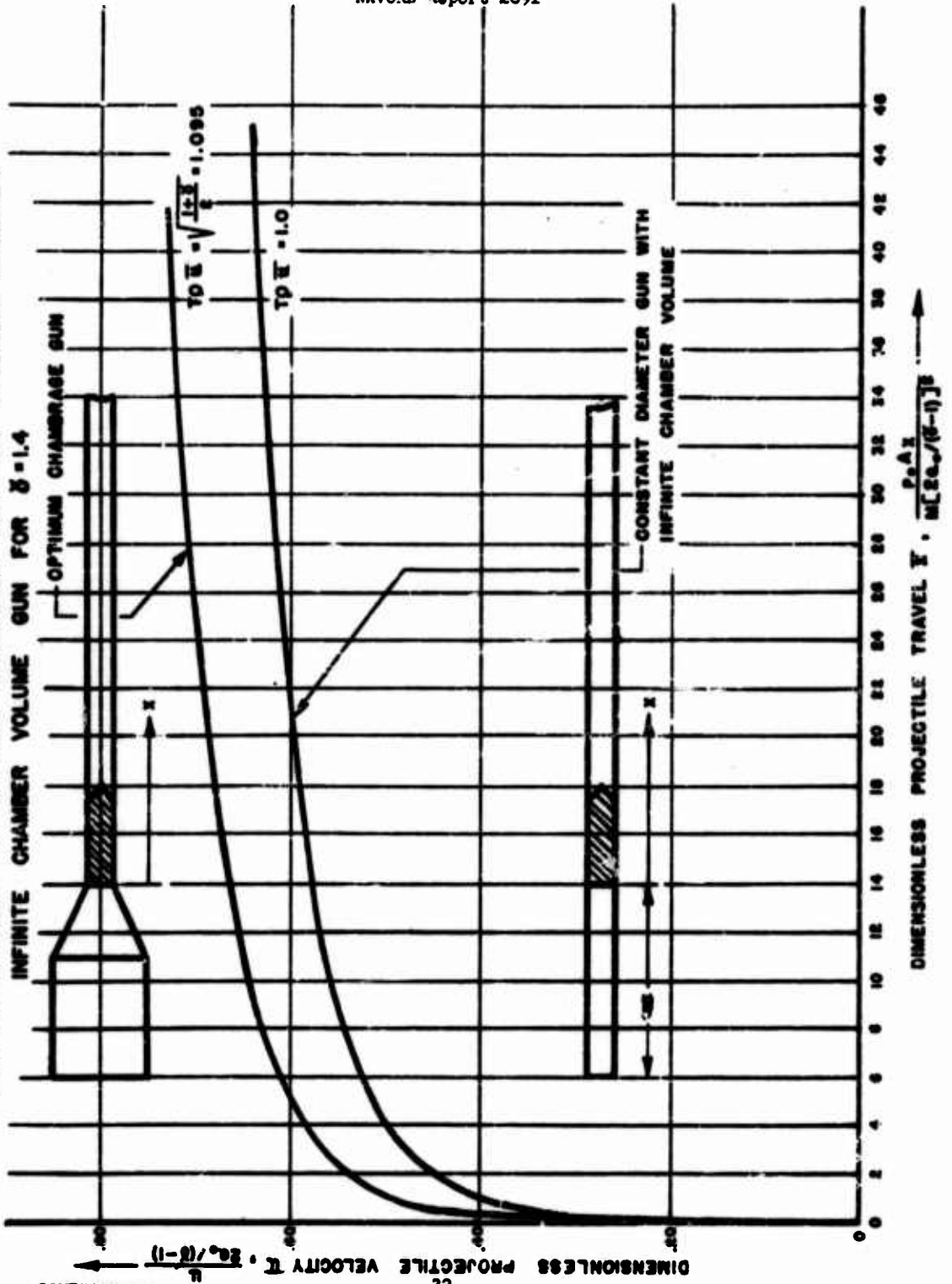FIG. 2 PORTION OF CHARACTERISTICS DIAGRAM FOR OPTIMUM CHAMBRAGE GUN

FIG. 3  COMPARISON OF OPTIMUM CHAMBRAGE GUN WITH CONSTANT DIAMETER INFINITE CHAMBER VOLUME GUN FOR $\delta = 1.4$

FIG. 4 COMPARISON OF OPTIMUM CHAMBRAGE GUN WITH CONSTANT DIAMETER INFINITE CHAMBER VOLUME GUN FOR δ = 1.4 IN THE LOWER VELOCITY REGION
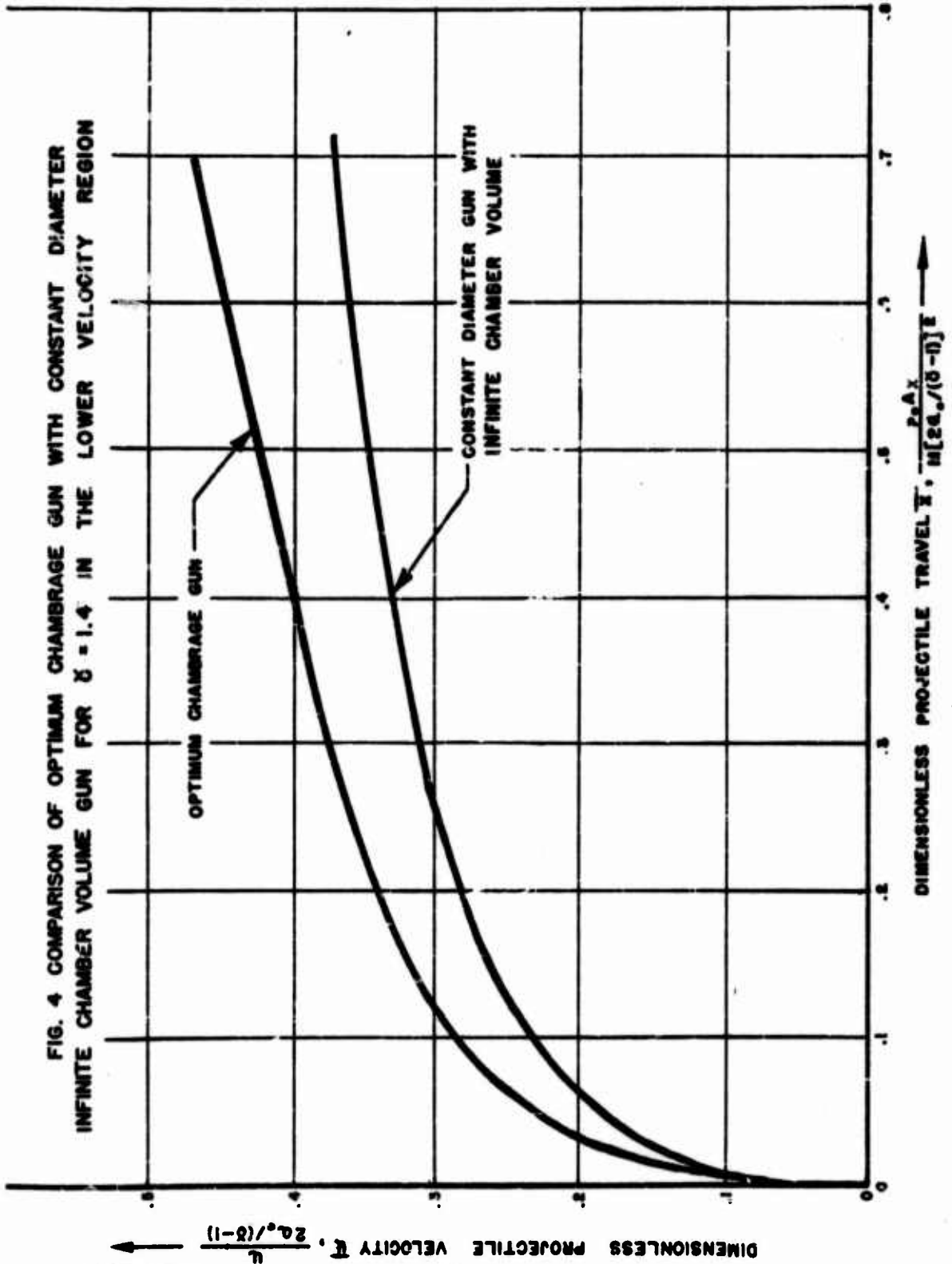
FIG. 5 COMPARISON OF THE PRESSURE BEHIND THE PROJECTILE FOR DIFFERENT PROJECTILE VELOCITIES FOR THE TWO DIFFERENT GUNS
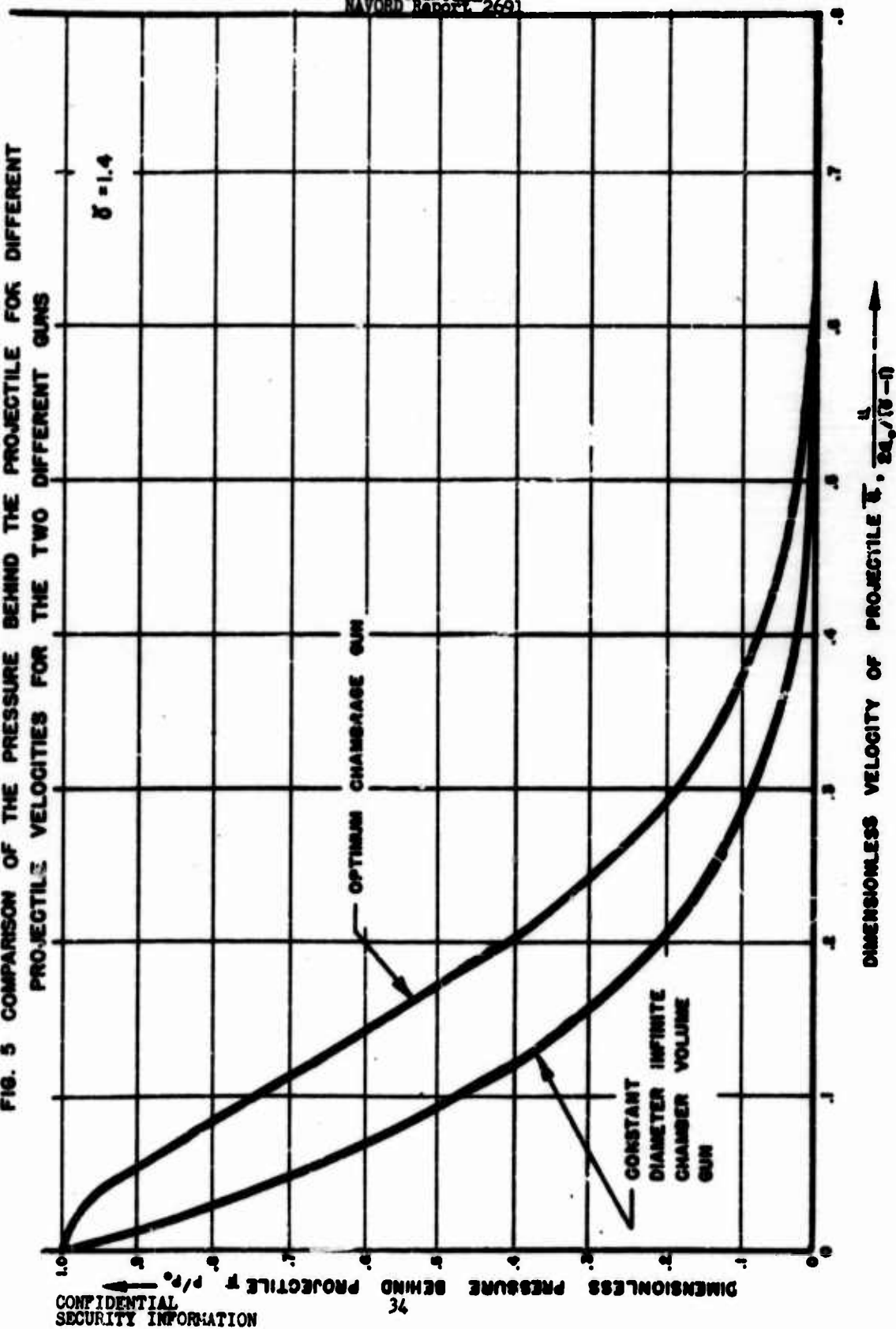
FIG. 6 COMPARISON OF OPTIMUM CHAMBRAGE GUN WITH CONSTANT DIAMETER INFINITE VOLUME GUN FOR $\delta = 1.25$

CURVE FOR CONSTANT DIAMETER INFINITE VOLUME GUN

APPROXIMATE CURVE FOR OPTIMUM CHAMBRAGE GUN

DIMENSIONLESS PROJECTILE TRAVEL $\bar{x}$, $\frac{P_0 A_0 x}{M[2a_0^2/(\delta-1)]}$

DIMENSIONLESS PROJECTILE VELOCITY $\bar{u}$, $\frac{u}{2a_0/(\delta-1)}$

FIG. 7   PRESSURE BEHIND A PROJECTILE MOVING AT CONSTANT VELOCITY IN AN OPTIMUM CHAMBRAGE GUN